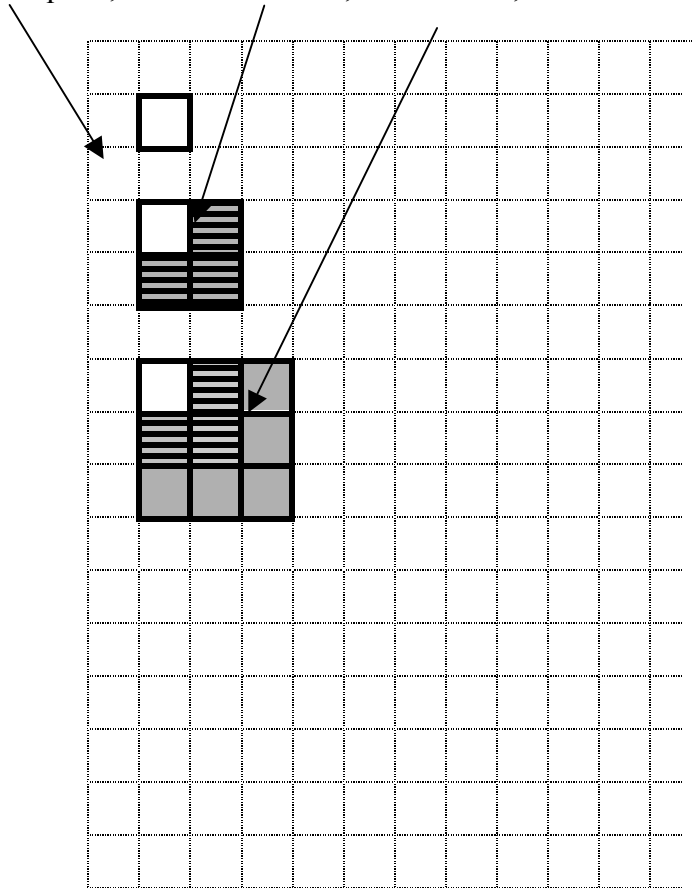

Odd Numbers

This problem gives you the chance to:

- work with shapes to make a number pattern
-

Kate makes a pattern of squares.

She starts with 1 square, then adds 3 more, then 5 more, and so on.



1 x 1 square

2 x 2 square

3 x 3 square

1. Draw the next shape in Kate's pattern.

2. How many new squares did you add? _____

3. What size square did you make? _____

The numbers of squares make a number pattern.

$$1 = 1 \times 1 = 1$$

$$1 + 3 = 2 \times 2 = 4$$

$$1 + 3 + 5 = 3 \times 3 = 9$$

4. Write the next two lines of the number pattern.

5. Use the number pattern to total the numbers.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

Show your work.

6. Write down the number pattern that gives a total of 169.

Explain your work.

Odd Numbers	Rubric	
The core elements of performance required by this task are: • work with shapes to make a number pattern Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. Draws a correct shape.	1	1
2. Gives correct answer: 7	1	1
3. Gives correct answer: 4 x 4 Accept 16	1	1
4. Writes correct lines: $1 + 3 + 5 + 7 = 4 \times 4 = 16$ $1 + 3 + 5 + 7 + 9 = 5 \times 5 = 25$	1 1	 2
5. Gives correct answer: 100 and Shows correct work = 10×10	1	1
6. Gives correct answer: $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$ and Gives correct explanation such as: $169 = 13^2$ so the number pattern contains the sum of 13 odd numbers>	1	1
Total Points		7

Odd Numbers

Work the task. Look at the rubric. What are some of the important elements of the pattern that you want students to notice? _____

Were your students able to draw the next figure in the pattern?

How did students describe the size of the square? Did they continue the pattern of using dimensions (4×4), did they use the total squares (16), or did they use some other numbers?

Look at the work in part 4, continuing the number pattern. How many of your students:

- Successfully completed both lines? _____
- Only attempted one line? _____
- Only showed the addition and total? _____
- Did not use consecutive odd numbers but got the correct total? _____

When doing pattern work in class, are students asked to describe in detail how the pattern is growing? Do they talk about the significant attributes of the pattern? Do they talk about the connections between the numbers in the numerical patterns and the attributes of the geometric pattern? How often are students asked to make connections between representations?

Now look at the work in part 5. While most students were able to get the correct total, many did not have strategies to simplify the process. How many of your students:

- Used a pattern (like thinking about groups of 20)?
- Counted the number of addends (10) and then knew that the total was 10×10 ?
- Chunked out small groups at a time looking for convenient numbers?
- Chunked out small groups at a time in order?

What opportunities do students have to practice mental math and think about strategies or patterns for simplifying the work? How might mental math help students develop their number sense and understanding of place value and number properties?

How does a strong number sense and understanding of number properties lay the foundation for understanding algebra?

Finally look at the work in part 6. How many students could:

- Write the expression of 13 consecutive odd numbers to total 169?
- How many students thought about finding the square root to help them with the pattern (13×13)?
- How many students did not give any explanation?
- How many students just added on from the previous answer without thinking about the attributes of the pattern?
- How many students gave number sentences totaling 169 that did not relate to the odd number pattern?

Did students use mathematical language, such as consecutive numbers, odd numbers, and square numbers, when discussing their strategies?

How can we provide students more opportunities to use academic language for a purpose? How does having academic language to look at different types of numbers help shape or contribute to your thought processes? How does it contribute to the types of patterns or attributes that you notice?

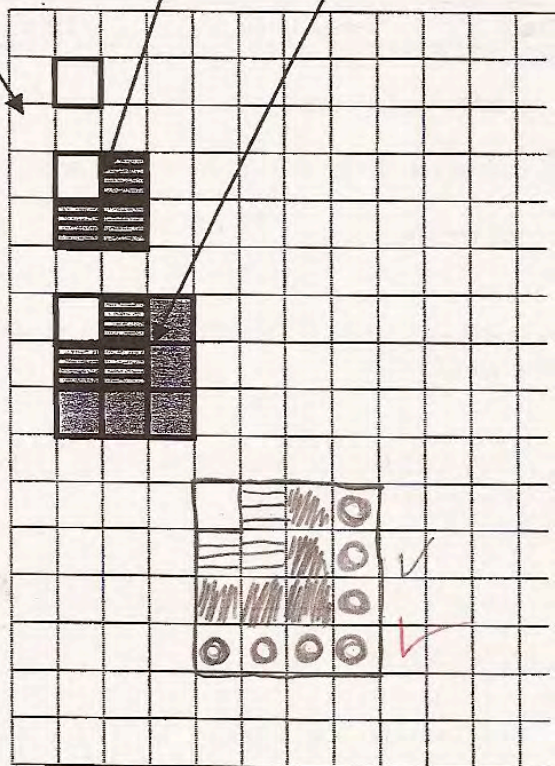
Looking at Student Work on Odd Numbers

Student A is able to see and extend the visual pattern. The student recognized the pattern is made up of two parts: the list of consecutive odd numbers and the square numbers. Notice that in part 6 the student is able to connect the number of consecutive addends to the number being squared.

Student A

Kate makes a pattern of squares.

She starts with 1 square, then adds 3 more, then 5 more, and so on.



1 x 1 square

2 x 2 square

3 x 3 square

4 x 4 square

1. Draw the next shape in Kate's pattern.

2. How many new squares did you add?

7 ✓

3. What size square did you make?

4 x 4 square ✓

1
1
1
1
1

Student A, part 2

The numbers of squares make a number pattern.

$$1 = 1 \times 1 = 1$$

$$1 + 3 = 2 \times 2 = 4$$

$$1 + 3 + 5 = 3 \times 3 = 9$$

4. Write the next two lines of the number pattern.

$$1 + 3 + 5 + 7 = 4 \times 4 = 16$$

$$1 + 3 + 5 + 7 + 9 = 5 \times 5 = 25$$

5. Use the number pattern to total the numbers.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

$$100$$

Show your work.

1	25	81
3	+11	+19
5	13	100
7	15	
9	17	
25	81	

6. Write down the number pattern that gives a total of 169.

Explain your work.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25$$

Because $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$ is 100, and there is 10 numbers in all, if I add the next three odd numbers, there will be 13 numbers, and $13 \times 13 = 169$.

Not all students with full marks noticed the same level of detail in the patterning. Student B does not connect the total in part 5 to dimensions of the square, 10 x 10. The student just continues the pattern of consecutive odd numbers until reaching the desired total of 169.

Student B

Show your work.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$	$16 + 48 + 36$
$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \quad \downarrow$	\downarrow
$4 \quad 12 \quad 20 \quad 28 \quad 36$	$64 + 36 = 100$

6. Write down the number pattern that gives a total of 169.

Explain your work.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 = 169$$

You just keep adding two numbers until you get the total you need.

Student C also needs to keep adding to the 100 from the previous total to find the numbers that total to 169. Notice that the student needs to look at every element in the pattern to solve for part 5. The student is not able to form a generalization to think about how the pattern grows independently of order.

Student C

5. Use the number pattern to total the numbers.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

Show your work.

$$1 = 1 \times 1 = 1$$

$$1 + 3 = 2 \times 2 = 4$$

etc....

$$1 + 3 + 5 + 7 + 9 = 5 \times 5 = 25$$

$$1 + 3 + 5 + 7 + 9 + 11 = 6 \times 6 = 36$$

etc....

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 8 \times 8 = 64$$

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 = 9 \times 9 = 81$$

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10 \times 10 = 100$$

$$\underline{100}$$

$$\begin{array}{r} 1 \\ +3 \\ +5 \\ +7 \\ +9 \\ +11 \\ +13 \\ +15 \\ +17 \\ +19 \\ \hline 100 \\ +21 \\ +23 \\ +25 \\ \hline 169 \end{array}$$

6. Write down the number pattern that gives a total of 169.

Explain your work.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 = 13 \times 13 = 169$$

I started out with 100 and added on 21, then 23, and then 25. Then it came up with 169.

$$\begin{array}{r} 100 \quad 10 \\ +21 \\ \hline 121 \quad 11 \\ +23 \quad 11 \\ \hline 144 \quad 12 \\ +25 \\ \hline 169 \quad 13 \end{array}$$

Student D is able to continue the pattern in part 4. The student does not look for a pattern to help with the addition in part 5, nor does the student think of 100 in connection with square numbers. In part 6, the student does not think about continuing the pattern of consecutive odd numbers, but just scrambles to find odd numbers to total 169. Do we continue asking probing questions in the classroom far enough to find out where student thinking breaks down? While in the early parts, the student seemed to have a good handle on the mathematics; the student could not extend that thinking to later parts of the task.

Student D

5. Use the number pattern to total the numbers.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

$$\underline{100}$$

Show your work.

$$\begin{array}{r} 1 \\ 3 \\ 5 \\ 7 \\ 9 \\ \hline 25 \end{array} + \begin{array}{r} 11 \\ 13 \\ 15 \\ 17 \\ 19 \\ \hline 75 \end{array} = 100$$

6. Write down the number pattern that gives a total of 169.

Explain your work.

~~$11 + 13 + 15 + 17 + 19 + 11 + 13 + 15 + 17 + 19 + 19 = 169$~~

$11 + 13 + 15 + 17 + 19 = 75 \quad \rightarrow \quad = 150 + 19 = 169$

~~$11 + 13 + 15 + 17 + 19 = 75$~~

Student E is able to pick up on some elements or attributes of the pattern, but needs to use a recursive rule to find the number being squared. This makes the rule more difficult to apply to later parts of the task. In parts 5 and 6 the student needs to rely on just arithmetic skills to crunch out the solutions.

Student E

4. Write the next two lines of the number pattern.

$1 + 3 + 5 + 7 = 4 \times 4 = 16$ the pattern is to add two and then multiply the number that's next by its self.

5. Use the number pattern to total the numbers.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

$$\underline{100}$$

Show your work.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$$

6. Write down the number pattern that gives a total of 169.

Explain your work.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 = 169$ I know that $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 100$ so I just added $21 + 23 + 25$ following the pattern above.

Being able to identify the attributes of the pattern and how those attributes contribute to the growth in the pattern is crucial to solving some of the later aspects of the task. Student F does not connect the addends to odd numbers and ignores the square numbers part of the pattern as irrelevant. In part 4 the student just tried to reach a total. The student relies on adding only 2 numbers at a time to cope with the string of numbers in part 5. The student doesn't have a pattern to use in solving part 6 and just adds 69 to the previous string of numbers.

Student F

$$1 + 1 \times 1 = 1$$

$$1 + 3 = 2 \times 2 = 4$$

$$1 + 3 + 5 = 3 \times 3 = 9$$

4. Write the next two lines of the number pattern.

10 + 4 = 14 X X 0 0

10 + 9 = 19 X X 0 0

5. Use the number pattern to total the numbers.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

100 ✓

Show your work.

$$\begin{array}{r} 9 \\ 7 \\ \hline 16 \end{array} + 5 = 21 + 3 = 24 + 1 = 25$$

$$\begin{array}{r} 25 \\ 11 \\ \hline 36 \end{array}$$

$$\begin{array}{r} 36 \\ 13 \\ \hline 49 \end{array}$$

$$\begin{array}{r} 49 \\ 15 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 64 \\ 17 \\ \hline 81 \end{array}$$

$$\begin{array}{r} 81 \\ 19 \\ \hline 100 \end{array}$$

6. Write down the number pattern that gives a total of 169.
Explain your work.

I got the numbers from Problem 5 and X (

just added 69 coz my answer was 100

so i just added 69 and it equals 169.

1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 69 X

Student G does not notice the square numbers in part 4, but only that two numbers are being multiplied together. The student is able to find a pattern of making 20's to simplify the addition in part 5. In part 6 the student adds in even numbers in an attempt to make 169. Without identifying a pattern, the answers to part 6 become random.

Student G

4. Write the next two lines of the number pattern.

$$1+3+5+7 = 2 \times 8 = 16 \quad \times$$

$$1+3+5+7+9 = 5 \times 5 = 25 \quad \checkmark \checkmark$$

0

0

1

5. Use the number pattern to total the numbers.

$$1+3+5+7+9+11+13+15+17+19$$

20
20
20
20
20

✓

100

1

Show your work.

$$1+3+5+7+9+11+13+15+17+19$$

$4+5 = 9$
 $9+7 = 16$
 $16+9 = 25$
 $25+11 = 36$
 $36+13 = 49$
 $49+15 = 64$
 $64+17 = 81$
 $81+19 = 100$

6. Write down the number pattern that gives a total of 169.

Explain your work.

$$1+4+5+7+8+10+12+14+15+17+19+20+21+24+24$$

31

x

x

0

+

$$1+2+3+11+2 = 16$$

a bunch of ...

$$\begin{array}{r} 21 \\ 34 \\ 65 \\ 51 \\ \hline 82 \\ 253 \end{array}$$

$$\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{array}$$

$1+2+3+4+5 = 15$
 $1+2+3+4+5+6+7+8+9+10 = 55$
 $1+2+3+4+5+6+7+8+9+10+11+12+13+14+15 = 105$
 $1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16+17+18+19+20 = 210$

$$\begin{array}{r} 12 \\ 13 \\ 14 \\ 15 \\ \hline 65 \end{array}$$

$$\begin{array}{r} 16 \\ 17 \\ 18 \\ \hline 51 \end{array}$$

$$\begin{array}{r} 19 \\ 20 \\ 21 \\ 22 \\ \hline 82 \end{array}$$

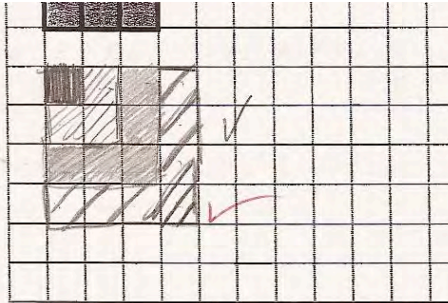
5

7

5

Student H does not see the relationship between the addends and the numbers being multiplied. In trying to add the string of numbers in part 5, the student uses number sense to find friendly combinations. However, the student's process is random so when he tries to extend the strategy into part 6 he loses sight of the idea of both consecutive and odd as elements of the pattern. *How do we help students develop the habits of mind of searching for order, defining or describing the parts of the pattern before solving a problem? What other habits of mind might be useful for this child?*

Student H



1. Draw the next shape in Kate's pattern.

2. How many new squares did you add? 4 ✓ X

3. What size square did you make? 8 inches X X

4. Write the next two lines of the number pattern.

$1+3+5+7=8 \times 8=16$ ✓ X 0
 $1+3+5+7+9=5 \times 5=25$ ✓ ✓ 1
 2

5. Use the number pattern to total the numbers.

$1+3+5+7+9+11+13+15+17+19$

Show your work.

$19+1=20$
 $3+7=10$
 $5+15=20$
 $11+9=20$
 $13+7=20$
100

100 ✓ 1
 1

6. Write down the number pattern that gives a total of 169.

Explain your work.

$1+3+5+7+9+11+13+15+17+19+19+1+15+5+6+9+3+6$ 1st I made 0
it to 100 then I thought up numbers that equal 20 then
when I did two numbers that equal 9 X 0

Student I focuses on the square numbers in the pattern, but does not notice the consecutive odd numbers that make up the addends.

Student I

$$1 + 3 + 5 = 3 \times 3 = 9$$

4. Write the next two lines of the number pattern.

$$3 + 4 + 1 + 8 = 4 \times 4 = 16 \quad \times \quad \times$$

$$4 + 3 + 2 + 10 + 5 = 5 \times 5 = 25 \quad \times \quad \times$$

5. Use the number pattern to total the numbers.

$$(1 + 3 + 5 + 7 + 9) + (11 + 13 + 15 + 17 + 19)$$

25 + 75

100 ✓ ✓

Show your work.

$$\begin{array}{r} 19 \\ 17 \\ 15 \\ 13 \\ +11 \\ \hline 75 \end{array} + \begin{array}{r} 9 \\ 11 \\ 13 \\ 15 \\ 17 \\ 19 \\ \hline 75 \end{array}$$

$$\begin{array}{r} 25 \\ +75 \\ \hline 100 \end{array}$$

6. Write down the number pattern that gives a total of 169.
Explain your work.

$$10 + 20 + 25 + 25 + 100 = 13 \times 13 = 169 \quad \times \quad \times$$

$\begin{array}{r} 2 \\ 17 \\ 19 \\ 21 \\ 23 \\ 24 \\ +25 \\ \hline 129 \\ +64 \\ \hline 193 \end{array}$	$\begin{array}{r} 3 \\ 13 \\ 11 \\ 09 \\ 07 \\ 05 \\ 03 \\ +01 \\ \hline 64 \end{array}$	$\begin{array}{r} 2 \\ 15 \\ 17 \\ 19 \\ +21 \\ \hline 62 \end{array}$	$\begin{array}{r} 2 \\ 01 \\ 03 \\ 07 \\ 09 \\ 11 \\ +13 \\ \hline 49 \end{array}$	$\begin{array}{r} 1 \\ 62 \\ +49 \\ \hline 111 \\ +23 \\ \hline 134 \\ +24 \\ \hline 158 \\ +25 \\ \hline 183 \end{array}$	$\begin{array}{r} 19 \\ 19 \\ \hline 38 \\ \times 13 \\ \hline 39 \\ +130 \\ \hline 169 \end{array}$
--	--	--	--	--	--

(3) 7

Student J struggles with addition. In order to cope with limited number fluency, the student simplifies or overlooks crucial elements of the pattern. *What classroom activities are appropriate at this grade level to help all students continue developing their number skills?*

Student J

$$1 + 3 + 5 = 3 \times 3 = 9$$

4. Write the next two lines of the number pattern.

$$1 + 3 + 5 + 7 = 16 \quad \checkmark 1$$

$$\quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \times \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \times 0$$

5. Use the number pattern to total the numbers.

$$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$$

$$91 \times \quad \times 0$$

Show your work.

$$1 + 3 = 4 \quad 4 + 5 = 9 \quad 9 + 7 = 16 \quad 16 + 11 = 27$$
$$27 + 13 = 40 \quad 40 + 15 = 55 \quad 55 + 17 = 72$$
$$72 + 19 = 91$$

6. Write down the number pattern that gives a total of 169.

Explain your work.

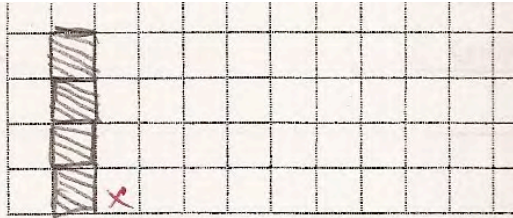
$$10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 + 10 \quad \times 0$$

$$+ 10 + 10 + 10 + 10 + 10 + 9 = 169$$

$$91 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 91$$
$$91 + 21 = 112 \quad 112 + 23 = 135 \quad 135 + 25 = 160$$
$$160 + 27 = 187 \quad 187 + 29 = 216$$

A few students struggled at just the basic level of seeing the visual pattern. Student K did not see the key attributes of the geometric or numeric patterns in the first 4 parts. The student is able to get the total in part 5, but loses points for not showing work.

Student K



1. Draw the next shape in Kate's pattern. X

2. How many new squares did you add? 4 X X

3. What size square did you make? 2x2 square X ^

4. Write the next two lines of the number pattern.

$1+3+12=4 \times 4=16$ X O

$1+3+21=5 \times 5=25$ X O

5. Use the number pattern to total the numbers.

$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$

100 ✓ ✓

Show your work.

^ ^ $\begin{array}{r} 85 \\ -1 \\ \hline 161 \end{array}$ O

6. Write down the number pattern that gives a total of 169.


Explain your work.


If you total of 169 it equals to 25 to get the total O


of 169 of it. ^ ^ X O

Student L does not notice the physical attributes of the pattern. Look at the responses for parts 1, 2, and 3. What aspects of the diagram or prompt might the student be paying attention to? Where might these answers be coming from? What types of help do students L and K need prepare them to work on grade level tasks? What is feasible within the classroom? What other types of help might be available at your school?

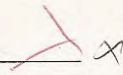



Student L

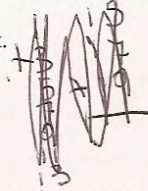
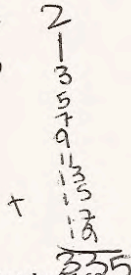
1. Draw the next shape in Kate's pattern. 




2. How many new squares did you add? 4 

3. What size square did you make? 2x2 square 

$1 + 3 = 2 \times 2 = 4$
 $1 + 3 + 5 = 3 \times 3 = 9$

4. Write the next two lines of the number pattern. ???  
 

5. Use the number pattern to total the numbers.
 $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$
 Show your work.  

6. Write down the number pattern that gives a total of 169.
 Explain your work.
I added random odd numbers  
and added them to get 169.

~~9 + 19 + 13 + 11 + 15 + 17 + 19~~
 $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 + 31 + 33 + 35 + 37 + 39 + 41 + 43 + 45 + 47 + 49 = 169$

Student Task	Work with shapes to make a number pattern.
Core Idea 3 Algebra and Function	<p>Understand relations and functions, analyze mathematical situations, and use models to solve problems involving quantity and change.</p> <ul style="list-style-type: none"> • Represent, analyze, and generalize a variety of functions including linear and simple exponential relationships. • Express mathematical relationships using expressions and equations.
Core Idea 1 Number and Operation	<ul style="list-style-type: none"> • Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with rational numbers.

Mathematics in this task:

- Ability to identify attributes of a geometric pattern and extend the pattern
- Connect square numbers to the dimensions of a physical square
- Recognize consecutive odd numbers and extend them
- Addition with a series of addends
- Recognize and extend a numeric pattern
- Generalize about the elements of a pattern in order to extend it to later stages

Based on teacher observations, this is what seventh graders knew and were able to do:

- Draw and extend a geometric pattern
- Quantify the dimensions of a square
- Count the number of squares added to the next figure in the pattern
- Add a sequence of numbers totaling 100

Areas of difficulties for seventh graders:

- Make connections between different representations of the same pattern
- Adding a large string of numbers
- Finding a connection between the number of addends and the dimensions of the square
- Using a pattern when the numbers become too large

MARS Test Task 2 Frequency Distribution and Bar Graph, Grade 7

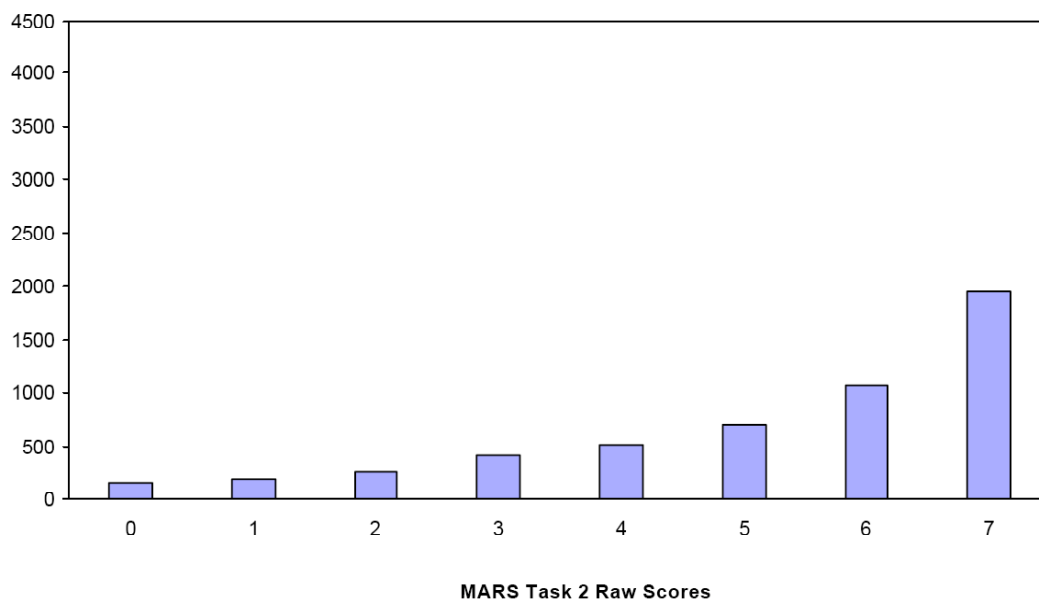
Task 2 - Odd Numbers

Mean: 5.26 StdDev: 1.95

Table 36: Frequency Distribution of MARS Test Task 2, Grade 7

Task 2 Scores	Student Count	% at or below	% at or above
0	157	3.0%	100.0%
1	183	6.5%	97.0%
2	255	11.3%	93.5%
3	412	19.2%	88.7%
4	514	29.0%	80.8%
5	698	42.3%	71.0%
6	1068	62.7%	57.7%
7	1957	100.0%	37.3%

Figure 45: Bar Graph of MARS Test Task 2 Raw Scores, Grade 7



The maximum score available on this task is 7 points.

The minimum score for a level 3 response, meeting standards, is 5 points.

Most students, 89%, could extend a geometric pattern, count the added squares, and give the dimensions of the square. Many students, 81%, could also add a sequence of numbers to equal 100. 71% of the students could extend and quantify the geometric pattern, extend one step of the numeric pattern and add numbers to equal 100. 37% of the students could meet all the demands of the task including extending the pattern of consecutive odd numbers to total 169. 3% of the students scored no points on this task. All the students in the sample with this score attempted the task.

Odd Numbers

Points	Understandings	Misunderstandings
0	All of the students in the sample with this score attempted the task.	Students had difficulty extending the geometric pattern. Some students made 2 x2 squares or made a vertical line of 4 squares. Some students colored in 1 extra square to the side of the 3 x 3 square. Some students thought only 3 or 4 new squares were added.
3	Students could extend a geometric pattern, count the added squares, and give the dimensions of the square.	Students struggled with adding the numbers to 100 in part 5 or did not show any work.
4	Students could complete part 1,2, and 3. Students could add numbers to total 100.	10% of the students only attempted one line of part 4. 10% added numbers that were not odd or not consecutive odd when extending the pattern. Some students did not continue to include the multiplication of square numbers as part of the pattern in 4.
5	Students with this score could usually complete the pattern for one of the lines in part 4.	Students with this score had difficulty with continuing the pattern in part 6 because of the size of 169 and the number of elements needed to make the pattern. Students looked for friendly numbers to make 169, such as $25 + 75 + 69$ or $10 + 10 + 10 + 10 \dots + 9$.
7	Students could extend a geometric and numeric pattern. Recognize the relationship between elements of the pattern and the total to extend the pattern given a total.	

Implications for Instruction

Students need to develop certain habits of mind when looking at patterns. They should routinely ask themselves questions, such as, “What stays the same? What changes? How are the numbers related to the geometric attributes? What are the important features of the numeric patterns? Are there any special types of numbers occurring in the pattern? Why is this outcome happening? How does one thing help me find something else? Can I find a rule that would help me extend the pattern without going one step at a time?” Students at this grade level should be reaching for how to make generalizations about solution strategies.

While it was not necessary to give a written generalization or rule to solve the elements in this task, students who noticed that the numbers were odd or odd consecutive numbers did better than students who just noticed that numbers were being added together. Some students could connect the number being squared with number of elements being added. Hopefully some students could connect the dimensions of the squares with the square numbers in the pattern.

At earlier grades students have been extending patterns and using totals to work backwards to find the place in the pattern. What is new and different for this grade level is to reach for generalizations and find ways to not draw and count every single item. Students need to be pushed to let go of the comfort of number crunching and try to find relationships, which in the long run will simplify their work and also allow them to use the pattern for any number.

Ideas for Action Research – Exploring Number Theory

While the format of this task is a familiar geometry pattern problem, the problem for students was not recognizing attributes in the numeric patterns. Many students did not pick up on the square numbers or the consecutive odd numbers. Try working some problems relying heavily on interesting number features. Consider trying the following problems with your class:

Sums of Consecutive Numbers

$$3 + 4 = 7$$

$$2 + 3 + 4 = 9$$

$$4 + 5 + 6 + 7 = 22$$

For each number from 1 to 35, find all the ways to write it as a sum of two or more consecutive numbers.

What can you discover about the sums of consecutive numbers? Find as many patterns as you can. Try to find at least 3.

Without doing any calculations, predict whether each of the following numbers can be made with 2 consecutive numbers, 3 consecutive numbers, 4 consecutive numbers, and so on.. Explain how you made your predictions.

- a. 45
- b. 57
- c. 62
- d. 75
- e. 80

Use your discoveries to come up with shortcuts for writing the same numbers as the sum of two or more consecutive numbers. Describe your shortcuts and tell how you used them to write the numbers as sums of consecutive numbers.

From: Fostering Algebraic Thinking, by Mark Driscoll

Other related problems from this source: *Something Nu*, *Stretching Problem*, *Differences of Squares*

Another related problem is: **King Arthur Problem**

Once upon a time, there were 19 knights seated around King Arthur's round table numbered 1 through 19. The king decided to use this method for choosing the one knight who would get to marry his beautiful daughter: starting with 1, he eliminated every second knight in order until only one knight remained. Thus, for 19 knights, he eliminated numbers 2, 4, 6, 8, 10, 12, 14, 16, 18, 1, 5, 9, 13, 17, 3, 11, 19 and 15. The remaining knight was knight number 7. The king decided to use the same method to determine the best mathematician in the realm. He called together 100 of the best thinkers and told them to sit at a circular table with 100 numbered seats. Of course YOU are one of these brilliant mathematicians. Which seat should you choose to be selected the best mathematician in the realm? Which seat should you choose if there are 500 seats? 1000 seats?