

into simpler parts, and using algorithms. Students at this grade level should develop a facility with doing operations with common fractions and mixed numbers as mental math activities.

5th grade

Task 3

Buttons

Student Task	Use a button arrangement pattern to describe, extend, and make generalization about its numeric pattern.
Core Idea 3 Patterns and Functions	<p>Understand patterns and use mathematical models such as algebraic symbols and graphs to represent and understand quantitative relationships.</p> <ul style="list-style-type: none"> • Describe and extend numeric patterns (3rd grade) • Represent and analyze patterns and functions using words (4th grade) • Investigate how a change in one variable relates to a change in a second variable

Buttons

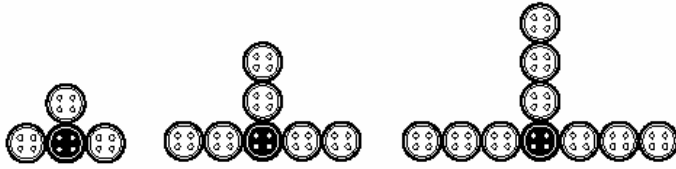
This problem gives you the chance to:

- describe, extend, and make generalizations about a numeric pattern

Gita plays with her grandmother's collection of black and white buttons.

She arranges them in patterns.

Her first 3 patterns are shown below.



Pattern 1

Pattern 2

Pattern 3

Pattern 4

1. Draw Pattern 4 next to Pattern 3.
2. How many **white** buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 _____

Pattern 6 _____

Explain how you figured this out.

3. How many buttons in all does Gita need to make Pattern 11?

Explain how you figured this out.

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

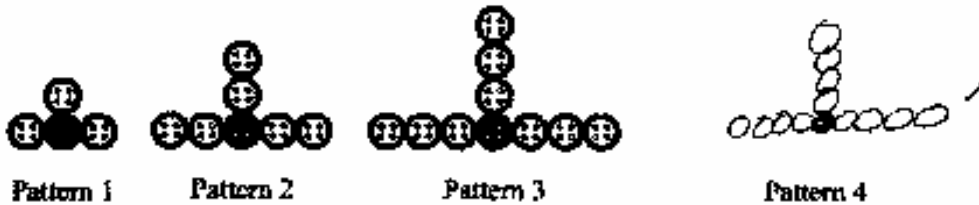
How do you know that she is **not** correct?

How many buttons does she need to make Pattern 24? _____

Buttons		Test 5 Form A Rubric	
The core elements of performance required by this task are: • describe, extend, and make generalizations about a numeric pattern Based on these, credit for specific aspects of performance should be assigned as follows:		Points	Section Points
1.	Draws correct Pattern 4.	1	1
2.	Gives correct answers as: Pattern 5 = 15 white buttons Pattern 6 = 18 white buttons Gives explanation such as: Pattern 5 has 3 rows of 5 white buttons = 15 white buttons and Pattern 6 has 3 rows of 6 white buttons = 18 white buttons. <i>Special case:</i> Gives answers 16 and 19 with correct explanations including black buttons.	1 1 1 2 s c	3
3.	Gives correct answer as: 34 Gives explanation such as: Pattern 11 has 3 rows of 11 white buttons = 33 white buttons and 1 black button. <i>Special case:</i> Gives answer as 33 with a correct explanation for the white buttons.	1 1 1 s c	2
4.	Gives explanation such as: Pattern 24 needs 3 rows of 24 white buttons = 72 white buttons and 1 black button, 73 buttons in all. <i>Accept alternative correct explanations such as: 69 is divisible by 3, so it cannot be correct.</i> Gives correct answer as: 73	1 1	2
Total Points			8

Looking at Student Work – Buttons

Many students at this grade level are able to see the pattern and form a generalization in words or in number algorithms. These generalizations could easily be converted to algebraic symbols at later grade levels. Student A has a nice description of how the pattern grows and an algorithm for find the total number of buttons in part 4.



1. Draw Pattern 4 next to Pattern 3.

2. How many white buttons does Gita need for Pattern 5 and Pattern 6?

Pattern 5 15 buttons Pattern 6 18 buttons

Explain how you figured this out.

For each pattern there is the same number of white
buttons...on 3 sides of the black button, so you count
the number of buttons on one side of the black buttons and
multiply that by three

3. How many buttons in all does Gita need to make Pattern 11?

34

Explain how you figured this out.

I did the same process in the problem above
but I added 1 to my answer for the black
button.

Student A, part 2

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is not correct?

Because if you follow the pattern of multiplying 3 to the number of buttons on one side and plus one for black button it doesn't equal 69.

How many buttons does she need to make Pattern 24? 73 buttons

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ + 1 \\ \hline 73 \end{array}$$

For further examples of making generalizations and algorithms, look at the work of Student B and C.

Student B

How do you know that she is not correct?

I know because $(24 \times 3) + 1$ does NOT equal 69.

How many buttons does she need to make Pattern 24?

73

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 + 1 = 73 \end{array}$$

Student C

Explain how you figured this out.

There are 11 white buttons on each line. So $11 \times 3 = 33$. Then plus 1
one black bead which is 34. ✓

$$33 + 1 = 34 \text{ buttons.}$$

How do you know that she is not correct?

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ + 1 \text{ black button} \\ \hline 73 \end{array}$$

While Student D does a thorough explanation of the growing pattern and finding her answers in part 2 and 3, the explanation about multiplying is unclear in part 4. However the work at the top of page shows a understanding of the process for finding the number of buttons in pattern 24.

Student D

Explain how you figured this out.

I figured this out because in pattern four we
put 4 white buttons in each side and 4 buttons in the
top. so I need 6 for each side in patterns and 6 on top.
and I need 6 on side in pattern 6 and plus 6 on top. ✓

3. How many buttons in all does Gita need to make Pattern 11? ✓

$$34 \checkmark$$

Explain how you figured this out.

I figured this because $3 \times 11 = 33$ plus
1 black button = 34. ✓

Student D, part 2

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

140
X 23
13

How do you know that she is not correct?

I know she is not correct because
she didn't multiply. x x

How many buttons does she need to make Pattern 24?

73 ✓ ✓

Student E has a very good generalization of the process for finding the total buttons in pattern 24. Unfortunately the student does not seem to use it to get the final answer.

Student E

4. Gita thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is not correct?

Twenty-four buttons are on each side of
the diamond button and on top. Add it all
together and get your answer.

How many buttons does she need to make Pattern 24?

113 X X 0

Some students can find the answer or come close to a solution, but use strategies that are inefficient, tedious, and may lead to errors. Student F uses a drawing and counting strategy. Student G does not see the complete pattern and solves for the white buttons every time. Student G uses repeated addition, which is a correct process, but makes a calculation error. Student H uses graphing, but because of the inaccuracies of the graph, the student's answer is incorrect.

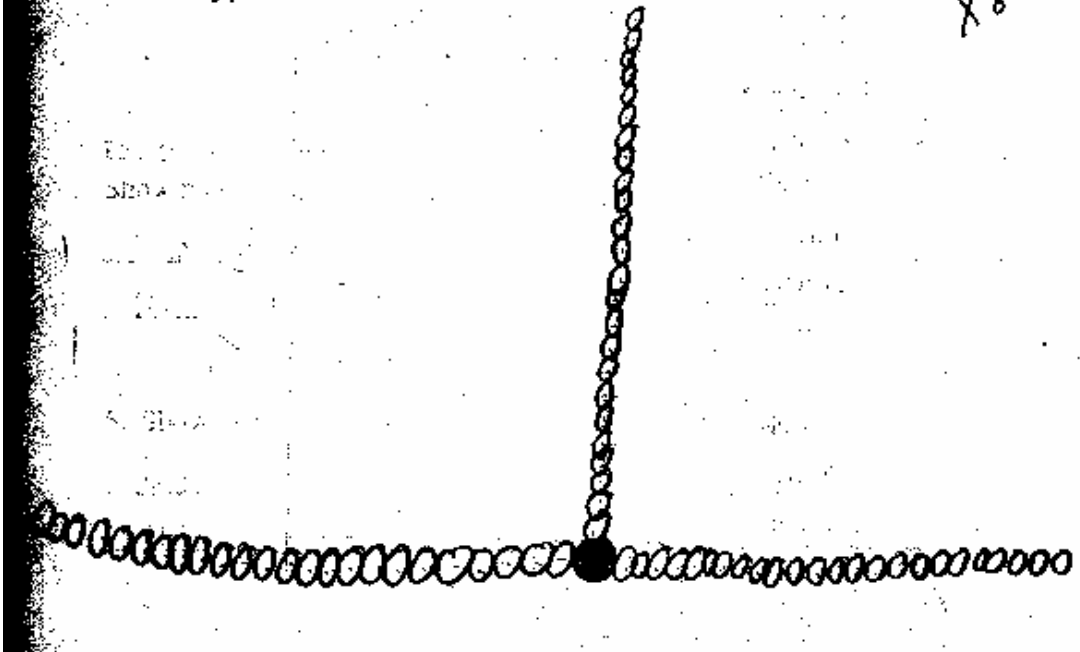
Student F

41. ~~Chia~~ thinks she needs 69 buttons in all to make Pattern 24.

How do you know that she is not correct?

because on each side of
the pattern it has the same
number

How many buttons does she need to make Pattern 24? 72 buttons



Student H

On the grid that I barely made ^{^C}
the answer is pattern 24 has 67. Pattern ^{67 has 2}
25 has 70 so that means the answer is ^{67 has 2}
How many buttons does she need to make Pattern 24? ⁶⁷ D

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19

0 1 2 3 4 5 6 7 8 9

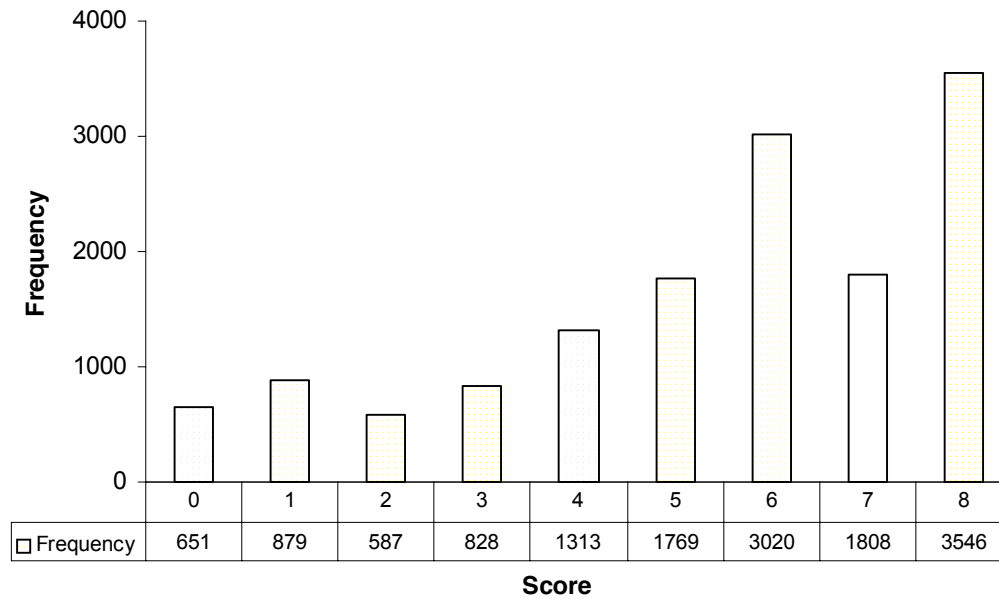
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Teacher Notes:

Grade 5 – Buttons

Buttons

Mean: 5.40, S.D.: 2.36



Score:	0	1	2	3	4	5	6	7	8
% < =	4.5%	10.6%	14.7%	20.4%	29.6%	41.9%	62.8%	75.4%	100.0%
% > =	100.0%	95.5%	89.4%	85.3%	79.6%	70.4%	58.1%	37.2%	37.2%

The maximum score available for this task is 8 points.

The cut score for a level 3 response is 4 points.

Most students (about 89%) could draw pattern 4 and give the correct number of buttons for pattern 5 and pattern 6. Many students (about 80%) could draw and extend the pattern and explain in words how the patterned worked. More than half the students (about 60%) could draw the pattern, extend the pattern for 5 and 6 and explain how it grew, find and explain the number of white buttons for pattern 11, and could find the total buttons for pattern number 24. Almost 26% of the students met all the demands of the task. About 5% of the students scored no points on this task.

Buttons

Points	Understandings	Misunderstandings
0	Most students attempted this task.	
3	Students could make a drawing of the next pattern in a sequence. They could also find the number of white buttons for pattern 5 and 6.	Students had difficulty explaining how they figured out the white buttons in the pattern. They said things like, "I added", without specifying what or why.
6	Students could find and extend a pattern. To find a solution in part 4,	They did not convert from white buttons to finding the pattern for all buttons in part 3 and 4. 37% of all the students multiplied 24 by 3 in part 4, but did not add on the black button.
7	31% of all students used the expression of 24 times 3 plus 1 to solve for part 4. Another 7% used $24+24+24+1$.	Some students used inefficient strategies, which may have led to errors. 13% drew a picture. 8% made a table and extended it, 3% just kept adding 3, and some students drew a graph.
8	Students could continue a pattern using a drawing. They could also find rules for extending it without drawing.	

Based on teacher observations, this is what fifth grade students know and are able to do:

- Continue a pattern using pictures and numbers
- Explain how a pattern grows and use that algorithm to solve for larger numbers in the pattern

Areas of difficulty for fifth graders, fifth grade students struggled with:

- Distinguishing between part of a pattern and the whole pattern
- Explaining a pattern in words

Teacher notes:

Questions for Reflection on Buttons:

- What are some rich problems that your students have done this year? What are some good resources for pattern problems?
- Do you ask questions like: “What stays the same?” and “What changes?” to help students develop the ability to form generalizations?
- Do students have opportunities to connect their number sentences to geometric patterns and share how they visualize the growth pattern?

Look carefully at your student work. What strategies did they use in Part 4?

Draw a picture	24×3 or $24+24+24$	$24 \times 3 + 1$ or $24+24+24+1$	Extend a table	Repeated addition of 3	Used a doubling strategy from a previous part of the problem

Teacher Notes:

Instructional Implications:

Fifth grade students need more experiences that require them to move beyond drawing the next figure in the pattern so that they analyze the pattern and represent the growth numerically. Fifth graders need to move beyond thinking about “what comes next?” to thinking about the problem as a whole: this involves generalizing what is happening with the growth, but need not necessarily involve variables or algebraic equations. Being able to see what remains the same and what changes in a pattern helps students develop algebraic thinking and the ability to make a generalization. Asking questions about how the pattern changes helps students to move beyond counting and drawing strategies to rules that will solve for any number in the pattern.