Key Concepts

Adding and Subtracting Fractions

Objective To learn how to add and subtract fractions with unlike denominators, using the method of common denominators.

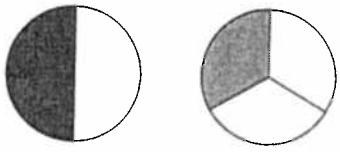
Note to the Teacher *This lesson contains the most complicated operations on fractions. Provide students with plenty of conceptual work with sketches (such as pizzas) and manipulative materials before introducing the algorithms.*

Using Models to Add Fractions with Unlike Denominators

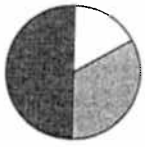
When adding fractions with unlike denominators, we cannot just add the numerators the way we do when the fractions have like denominators.

Example 1 **What is $\frac{1}{2} + \frac{1}{3}$?**

Solution In order to find this sum, we first need to rewrite both fractions so they have the same denominator. This means we must replace each fraction with an equivalent fraction having a different denominator. In order to motivate this process to your students, have them think about this problem using portions of a pizza. First, draw a separate model for each of the fractions.



The addition is modeled by taking the two parts together as a portion of a whole pizza.



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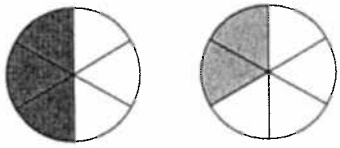
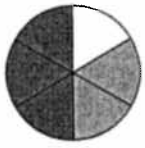
It is not easy to visually determine what fraction of the whole pizza is represented by the shaded portion of the model. However, suppose we divided each pizza into 2×3 or 6 equal parts.



Notice that each of the smaller parts we have created is one sixth of the pizza. Now when we merge the portions of the pizza, we get the model below.

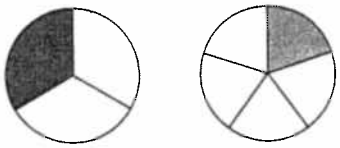


We can see that the shaded portion of the model consists of five pieces, each of which is one sixth of the pizza. In other words, the shaded portion of the pizza represents the fraction $\frac{5}{6}$, so

$$\frac{1}{2} + \frac{1}{3} = \frac{5}{6}.$$

Example 2 **What is $\frac{1}{3} + \frac{1}{5}$?**

Solution Represent each of the fractions $\frac{1}{3}$ and $\frac{1}{5}$ as portions of a pizza.



When we merge these two portions, we get this picture.

To determine what fraction is modeled by the combined shaded regions in the figure above, divide each pizza into 3×5 or 15 equal parts.

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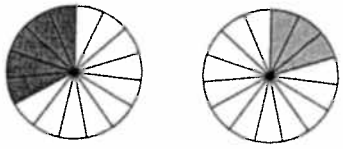
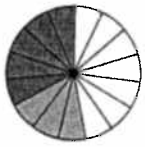


Each of the smaller parts we have created represents one fifteenth of the pizza. The merged version of these portions is shown below.

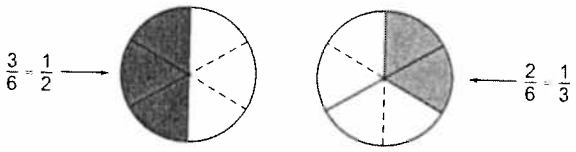


The shaded portion of the model above consists of eight pieces, each of which is one fifteenth of the pizza. The shaded portion of the model represents $\frac{8}{15}$, so

$$\frac{1}{3} + \frac{1}{5} = \frac{8}{15}.$$

The Algorithm and Why It Works

Let's think about the problem of adding $\frac{1}{2}$ and $\frac{1}{3}$ (Example 1). When we divided the pizza portion corresponding to $\frac{1}{2}$ into three equal pieces, each of the three pieces represented one sixth of the pizza. The model showed that $\frac{3}{6}$ and $\frac{1}{2}$ are **equivalent fractions**. In the same way, when the pizza portion representing $\frac{1}{3}$ was divided into two equal pieces, both pieces represented one sixth of the pizza. This showed that $\frac{2}{6}$ and $\frac{1}{3}$ are equivalent fractions.



To do this step without using the pizza models, we replace each fraction by an equivalent fraction so that the two fractions have the same denominators.

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$$\begin{array}{lccc} & \frac{1}{2} & + & \frac{1}{3} \\ \text{equivalent fractions} & \downarrow & & \downarrow \text{ equivalent fractions} \\ & \frac{3}{6} & + & \frac{2}{6} \end{array}$$

We can easily add these equivalent fractions because they have like denominators.

$$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6}$$
$$= \frac{3+2}{6} \text{ or } \frac{5}{6}$$

Note to the Teacher *Stress to students that when they multiply both numerator and denominator of a fraction by the same number, the result is an equivalent fraction.*

Example 3 **Find $\frac{1}{3} + \frac{1}{5}$ using equivalent fractions with the same denominator.**

Solution First, find equivalent fractions for both $\frac{1}{3}$ and $\frac{1}{5}$ that have the same denominator. Multiply both the numerator and denominator of $\frac{1}{3}$ by 5 $\left(\text{the denominator of } \frac{1}{5}\right)$.

$$\frac{1}{3} = \frac{1 \times 5}{3 \times 5} = \frac{5}{15}$$

So $\frac{5}{15}$ is equivalent to $\frac{1}{3}$. Now multiply both the numerator and denominator of $\frac{1}{5}$ by 3 $\left(\text{the denominator of } \frac{1}{3}\right)$.

$$\frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}$$

So $\frac{3}{15}$ is equivalent to $\frac{1}{5}$. Now we can add the equivalent fractions.

$$\frac{1}{3} + \frac{1}{5} = \frac{5}{15} + \frac{3}{15}$$
$$= \frac{5+3}{15} \text{ or } \frac{8}{15}$$

Note to the Teacher *Stress that finding equivalent fractions with like denominators can be accomplished by multiplying both the numerator and denominator of each fraction by the denominator of the other fraction.*

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Example 4 **Find the sum $\frac{2}{3} + \frac{3}{7}$.**

Solution Multiply both the numerator and denominator of $\frac{2}{3}$ by 7 (the denominator of $\frac{3}{7}$).

$$\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$$

Multiply both the numerator and denominator of $\frac{3}{7}$ by 3 (the denominator of $\frac{2}{3}$).

$$\frac{3}{7} = \frac{3 \times 3}{7 \times 3} = \frac{9}{21}$$

Now replace each original fraction with its equivalent fraction and add.

$$\frac{2}{3} + \frac{3}{7} = \frac{14}{21} + \frac{9}{21}$$

$$= \frac{14 + 9}{21} \text{ or } \frac{23}{21}$$

The fraction $\frac{23}{21}$ is an **improper fraction**, which is a fraction whose numerator is greater than its denominator. Since 21 goes into 23 once with a remainder of 2, the sum can also be written as a mixed number. So,

$$\frac{2}{3} + \frac{3}{7} = \frac{23}{21} \text{ or } 1\frac{2}{21}.$$

Here is an algorithm for adding fractions with unlike denominators.

Algorithm for Adding Fractions with Unlike Denominators

1. Multiply both the numerator and denominator of each fraction by the denominator of the other fraction. The resulting equivalent fractions will have the same denominator.

$$\begin{array}{ccc} \frac{1}{9} & + & \frac{2}{7} \\ \downarrow & & \downarrow \\ \frac{1 \times 7}{9 \times 7} & + & \frac{2 \times 9}{7 \times 9} \\ \downarrow & & \downarrow \\ \frac{7}{63} & + & \frac{18}{63} \end{array}$$

2. Add the equivalent fractions by adding their numerators and retaining the like denominator.

$$\frac{7}{63} + \frac{18}{63} = \frac{25}{63}$$

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Example 5 Find $\frac{2}{9} + \frac{3}{11}$.

Solution **Step 1** Multiply both the numerator and denominator of $\frac{2}{9}$ by 11, and multiply both the numerator and denominator of $\frac{3}{11}$ by 9.

$$\frac{2}{9} = \frac{2 \times 11}{9 \times 11} = \frac{22}{99} \quad \text{and} \quad \frac{3}{11} = \frac{3 \times 9}{11 \times 9} = \frac{27}{99}$$

Step 2 Add the equivalent fractions.

$$\frac{22}{99} + \frac{27}{99} = \frac{22 + 27}{99} \text{ or } \frac{49}{99}$$

So, $\frac{2}{9} + \frac{3}{11} = \frac{49}{99}$.

Subtracting Fractions with Unlike Denominators

The algorithm for subtracting one fraction from another when they have unlike denominators is very similar to the algorithm for adding fractions with unlike denominators.

Algorithm for Subtracting Fractions with Unlike Denominators	1. Multiply both numerator and denominator of each fraction by the denominator of the other fraction. The resulting equivalent fractions will have the same denominator. 2. Subtract the equivalent fractions by subtracting the numerators and retaining the like denominator.

Example 6 What is the value of $\frac{2}{3} - \frac{1}{5}$?

Solution **Step 1** Multiply both the numerator and denominator of $\frac{2}{3}$ by 5, and multiply both the numerator and denominator of $\frac{1}{5}$ by 3.

$$\frac{2}{3} = \frac{2 \times 5}{3 \times 5} = \frac{10}{15} \quad \text{and} \quad \frac{1}{5} = \frac{1 \times 3}{5 \times 3} = \frac{3}{15}$$

Step 2 Replace each original fraction with its equivalent fraction. Subtract the equivalent fractions by subtracting numerators and retaining the denominator.

$$\frac{10}{15} - \frac{3}{15} = \frac{10 - 3}{15} \text{ or } \frac{7}{15}$$

So, $\frac{2}{3} - \frac{1}{5} = \frac{7}{15}$.

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Example 7 Find $\frac{5}{7} - \frac{2}{3}$.

Solution **Step 1** $\frac{5}{7} = \frac{5 \times 3}{7 \times 3} = \frac{15}{21}$ and $\frac{2}{3} = \frac{2 \times 7}{3 \times 7} = \frac{14}{21}$

Step 2 $\frac{15}{21} - \frac{14}{21} = \frac{15 - 14}{21}$ or $\frac{1}{21}$

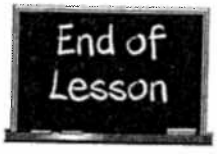
So, $\frac{5}{7} - \frac{2}{3} = \frac{1}{21}$.

We have discussed procedures for adding and subtracting fractions, as well as the reasoning behind them. It is very important to keep the reasoning in mind, since it allows students to interpret practical problems correctly, as well as to recall the procedures. One particularly common error made by students can be avoided by keeping the meaning of the fractions in mind. When adding fractions, some students make the mistake of adding denominators as well as numerators. For example,

$$\frac{2}{3} + \frac{4}{5} = \frac{2 + 4}{3 + 5} \text{ or } \frac{6}{8}.$$

By recognizing that each of the fractions being added is greater than $\frac{1}{2}$, students will see that an answer less than 1 is unreasonable.

Note to the Teacher *Divide your class into two groups. Give each group several addition and subtraction problems involving fractions, some of which are solved correctly and others that are solved incorrectly. Have each group discuss which problems are solved correctly and which are not, giving reasons for their decisions. Have each group present their findings to the rest of the class.*



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